In complete sentences, using proper English and mathematical notation, state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

IF f IS CONTINUOUS ON $[a,b]$
AND $g(x) = \int_a^x f(t)\,dt$
THEN $g'(x) = f(x)$ ON (a,b)

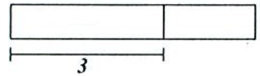
IF f IS CONTINUOUS ON $[a,b]$
AND $F'(x) = f(x)$ ON $[a,b]$
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

GRADED BY ME

IF F' IS CONTINUOUS ON $[a,b]$
THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

A rod is made of a mix of materials, so that its density is NOT constant along its length. **SCORE: _____ / 3 PTS**

Suppose the function $p(x)$ gives the linear density (in ounces per inch) of the rod x inches from its left end.



For example, if $p(3) = 2$, that means that if the rod were made entirely of the material that is 3 inches from the left end (the thin vertical sliver in the middle of the rod in the diagram), then each inch of that rod would weigh 2 ounces.

What is the meaning of the equation $\int_2^4 p(x)\,dx = 7$ in this situation ?

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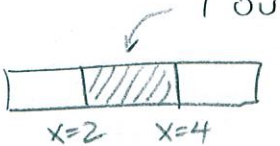
NOTES: **Your answer must use all three numbers from the equation, along with correct units.**
Your answer should NOT use "x", "$p(x)$", "integral", "antiderivative", "rate of change" or "derivative".

IF $w(x)$ = WEIGHT OF ROD FROM LEFT END
TO X INCHES FROM LEFT END

$w'(x) = p(x)$

SO $\int_2^4 p(x)\,dx = \int_2^4 w'(x)\,dx = w(4) - w(2) = 7 \frac{\text{OUNCES}}{\cancel{\text{INCH}}} \times \cancel{\text{INCH}}$

7 OUNCES = WEIGHT OF ROD
FROM 2 INCHES FROM LEFT END
TO 4 INCHES FROM LEFT END



Evaluate the following integrals, or explain why they can't be evaluated. ALL ITEMS ① POINT UNLESS OTHERWISE MARKED SCORE: _____ / 11 PTS

[a] $\int \frac{(5-2\sqrt{y})^2}{3y^2}\,dy$

$= \int \frac{25-20y^{\frac{1}{2}}+4y}{3y^2}\,dy$

$= \int \left(\frac{25}{3}y^{-2} - \frac{20}{3}y^{-\frac{3}{2}} + \frac{4}{3}y^{-1}\right)dy$

$= -\frac{25}{3}y^{-1} - \frac{20}{3}(-2)y^{-\frac{1}{2}} + \frac{4}{3}\ln|y| + C$

$= \underbrace{-\frac{25}{3}y^{-1} + \frac{40}{3}y^{-\frac{1}{2}} + \frac{4}{3}\ln|y|}_{1\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$

[b] $\int_{-e}^{e} \frac{(1+\ln|t|)^3}{t}\,dt$

DISCONTINUOUS @ $t=0$

FTC DOES NOT APPLY $\frac{1}{2}$

[c] $\int_{-2}^{2} \frac{5x}{\sqrt{1+x^6}}\,dx = 0$ $\frac{1}{2}$

CONTINUOUS $\frac{1}{2}$

$$\frac{5(-x)}{\sqrt{1+(-x)^6}} = -\frac{5x}{\sqrt{1+x^6}}$$

ODD

$\frac{1}{2}$

[d] $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin\theta\cos\theta}{\sqrt{1-\cos^4\theta}}\,d\theta$

$u = \cos^2\theta$; $\theta = \frac{\pi}{2} \longrightarrow u = 0$; $\theta = \frac{\pi}{4} \longrightarrow u = \frac{1}{2}$

$$\frac{du}{d\theta} = -2\cos\theta\sin\theta$$

$$-\frac{1}{2}du = \cos\theta\sin\theta\,d\theta$$

$$\int_{\frac{1}{2}}^{0} -\frac{1}{2}\frac{1}{\sqrt{1-u^2}}\,du$$

$$= -\frac{1}{2}\sin^{-1}u\Big|_{\frac{1}{2}}^{0}$$

$$= -\frac{1}{2}\left(0-\frac{\pi}{6}\right) = \frac{\pi}{12}$$

If $p(x) = \int_{x^3}^{5x} \sqrt{1+t^2}\, dt$, find $p'(2)$. **SCORE: _____ / 5 PTS**

$$p(x) = \int_{x^3}^{2} \sqrt{1+t^2}\,dt + \int_{2}^{5x} \sqrt{1+t^2}\,dt$$

$$= -\int_{2}^{x^3} \sqrt{1+t^2}\,dt + \int_{2}^{5x} \sqrt{1+t^2}\,dt$$

$$p'(x) = \frac{d}{d(x^3)}\left(-\int_{2}^{x^3}\sqrt{1+t^2}\,dt\right)\cdot\frac{dx^3}{dx} + \frac{d}{d(5x)}\int_{2}^{5x}\sqrt{1+t^2}\,dt\cdot\frac{d(5x)}{dx}$$

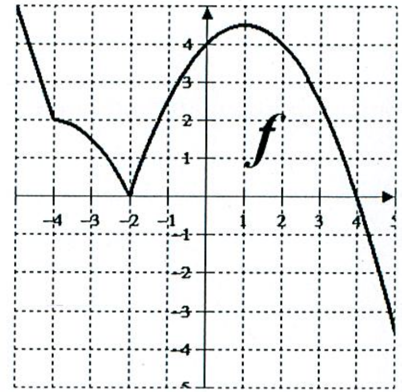
$$= \underbrace{-}_{1}\underbrace{\sqrt{1+(x^3)^2}}_{1}\cdot\underbrace{3x^2}_{1} + \underbrace{\sqrt{1+(5x)^2}}_{\frac{1}{2}}\cdot\underbrace{5}_{1}$$

$$p'(2) = -\sqrt{65}\cdot 12 + \sqrt{101}\cdot 5$$

$$= \underbrace{5\sqrt{101} - 12\sqrt{65}}_{\frac{1}{2}}$$

$g'(x) = f(x)$

Let $g(x) = \int_{-5}^{x} f(t)\, dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 6 PTS**



[a] Write "I UNDERSTAND THAT THE GRAPH SHOWS f, BUT THE QUESTIONS ASK ABOUT g".

[b] Find $g'(-4)$. **Explain your answer very briefly.**

$g'(-4) = \underbrace{f(-4) = 2}_{}$ 1

[c] Find all critical numbers of g. **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) = 0}_{1}$ @ $\underbrace{x = -2, 4}_{2}$ ← IF YOU GOT BOTH CORRECT ANSWERS, BUT ALSO ADDITIONAL INCORRECT ANSWERS, SUBTRACT 1 POINT PER INCORRECT ANSWER

[d] Find all intervals over which g is concave up. **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) \text{ INCREASING}}_{1}$ ON $\underbrace{(-2, 1)}_{1}$